Instructions: Choose EXACTLY 10 points to be graded. I will evaluate anything you turn in but you MUST indicate which 10 points you want to count towards your grade. You will not earn full credit if your choice is not clear even if all of your solutions are correct. You are free to ask me any questions and work with whomever you like. If you work with any one else or use any resources beyond the textbook and class notes, please let me know. *Quiz 6 in D2L.*

For computations: you do not need to obtain a single value as your answer—it is acceptable to submit an expression that could be plugged into a scientific calculator and give a value. That is, all integrals must be evaluated but not necessarily simplified beyond that.

1. (2 points) A chain lying on the ground is 15 m long and its mass is 100 kg. How much work is require to raise one end of the chain to a height of 12 m?
2. (3 points) A swimming pool is 20 ft wide and 40 ft long and its bottom is an inclined plane, the shallow end having a depth of 3 ft and the deep end, 9 feet (i.e., over the 40 ft length, the water becomes 6 ft deeper). If the pool is full of water, estimate the hydrostatic force on the shallow end, the deep end, one of the sides, and the bottom of the pool.
3. (2 points) A spring has a natural length of 20 cm. If a 25 N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm?
4. (2 points) Find the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$.
5. (2 points) For each condition below, give an example of a sequence $\{a_n\}$ with the desired property and justify that your sequence satisfies the given property.
   (a) Converges to 0
   (b) Converges but not to 0
   (c) Diverges and bounded
   (d) Diverges, unbounded, $\lim_{n\to\infty} a_n \neq \infty$ or $-\infty$
6. (4 points) Classify each of the following statements as "always true", "sometimes true", or "never true" and fully justify your choice (i.e., if you think the statement is sometimes true, give an example where the condition holds and an example where the condition does not hold but if you think the statement is always true or always false, prove your claim).
   (a) The sequence a_n converges to 0 and the series $\sum a_n$ converges.
   (b) The sequence a_n converges to 1 and the series $\sum a_n$ diverges.
   (c) The sequence $\{a_n + b_n\}$ satisfies $\lim_{n\to\infty} a_n + b_n = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$.
   (d) The series $\sum a_n$ and $\sum b_n$ converge and $\sum a_n - b_n$ diverges.
7. (2 points) Find the value of c such that

$$\sum_{n=1}^{\infty} e^{nc} = 10$$

8. Determine if the following series converge or diverge. If they converge, determine what they converge to.

   (a) (2 points) $\displaystyle\sum_{n=1}^{\infty} \frac{-2}{n^2+n}$

   (b) (2 points) $\displaystyle\sum_{n=1}^{\infty} \ln(n)$

   (c) (2 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{\pi^{n+3}}$

9. Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

   (a) (2 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

   (b) (2 points) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{1+n^2}$

   (c) (2 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-5)^n}{n!}$

   (d) (2 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

   (e) (2 points) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+2}{n}\right)$

   (f) (2 points) $\displaystyle\sum_{n=1}^{\infty} a_n$ where $\{a_n\}$ is the sequence defined recursively by $a_1 = 2$ and

   $$a_{n+1} = \left(\frac{5n+1}{4n+3}\right) a_n$$

   (g) (2 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

   (h) (2 points) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3 \cdot 4^n}{(-5)^{n+1}} + 7^{-n}\right)$

10. (2 points) How many terms should be added to estimate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ to within 0.01 of its true value?

11. (3 points) Emmy Noether is measuring the velocity in feet per second of a pie rolling on its side. Her data is recorded in the table below.

Time (s)	0	1	2	3	4	5
Velocity (ft/s)	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$	$\frac{1}{27}$

(a) Assuming the pattern continues, find a general expression for the velocity at n seconds. We will denote this velocity by v_n.

(b) Note that the distance traveled by the pie after n seconds can be approximated by $s_n = \sum_{i=0}^{n} v_i$. The total distance traveled by the pie is given by $\sum_{n=0}^{\infty} v_n$. After how many seconds is the pie within 0.1 feet of the total distance traveled?

(c) Let the number of seconds found in part (b) be n. Use s_n and the upper and lower bounds for the error to estimate the total distance the pie will travel.